



UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS  
General Certificate of Education Advanced Level

**MATHEMATICS**

**9709/53**

Paper 5 Mechanics 2 (M2)

**May/June 2011**

**1 hour 15 minutes**

Additional Materials: Answer Booklet/Paper  
Graph Paper  
List of Formulae (MF9)



**READ THESE INSTRUCTIONS FIRST**

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Where a numerical value for the acceleration due to gravity is needed, use  $10 \text{ m s}^{-2}$ .

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

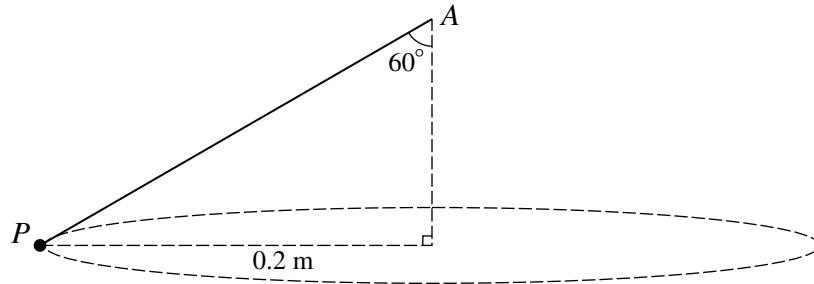
The total number of marks for this paper is 50.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **4** printed pages.



1



A particle  $P$  of mass  $0.4 \text{ kg}$  is attached to a fixed point  $A$  by a light inextensible string. The string is inclined at  $60^\circ$  to the vertical.  $P$  moves with constant speed in a horizontal circle of radius  $0.2 \text{ m}$ . The centre of the circle is vertically below  $A$  (see diagram).

(i) Show that the tension in the string is  $8 \text{ N}$ . [2]

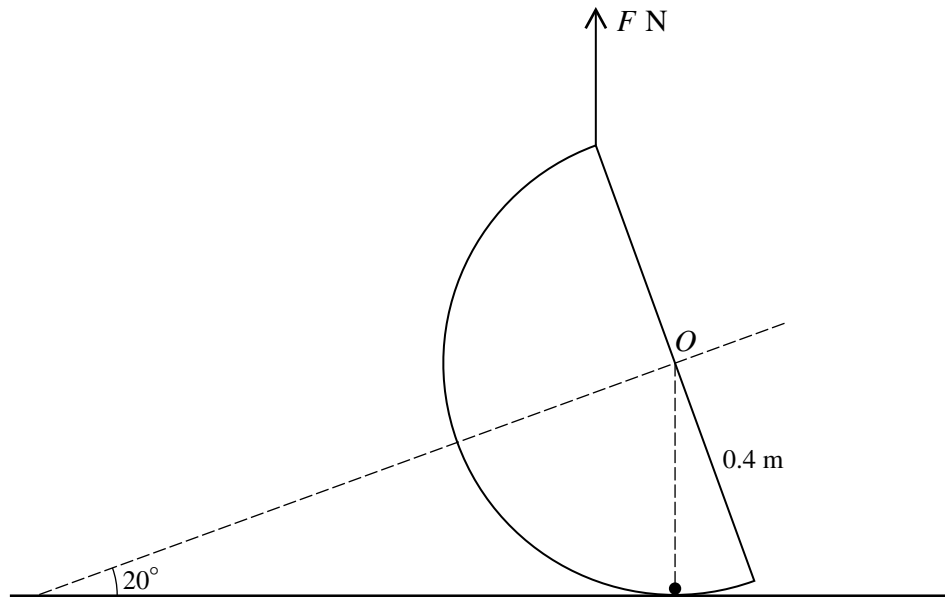
(ii) Calculate the speed of the particle. [2]

2 A stone is thrown with speed  $15 \text{ m s}^{-1}$  horizontally from the top of a vertical cliff  $20 \text{ m}$  above the sea. Calculate

(i) the distance from the foot of the cliff to the point where the stone enters the sea, [3]

(ii) the speed of the stone when it enters the sea. [3]

3



A smooth hemispherical shell, with centre  $O$ , weight  $12 \text{ N}$  and radius  $0.4 \text{ m}$ , rests on a horizontal plane. A particle of weight  $W \text{ N}$  lies at rest on the inner surface of the hemisphere vertically below  $O$ . A force of magnitude  $F \text{ N}$  acting vertically upwards is applied to the highest point of the hemisphere, which is in equilibrium with its axis of symmetry inclined at  $20^\circ$  to the horizontal (see diagram).

(i) Show, by taking moments about  $O$ , that  $F = 16.48$  correct to 4 significant figures. [3]

(ii) Find the normal contact force exerted by the plane on the hemisphere in terms of  $W$ . Hence find the least possible value of  $W$ . [3]

- 4 The ends of a light elastic string of natural length 0.8 m and modulus of elasticity  $\lambda$  N are attached to fixed points  $A$  and  $B$  which are 1.2 m apart at the same horizontal level. A particle of mass 0.3 kg is attached to the centre of the string, and released from rest at the mid-point of  $AB$ . The particle descends 0.32 m vertically before coming to instantaneous rest.

(i) Calculate  $\lambda$ . [4]

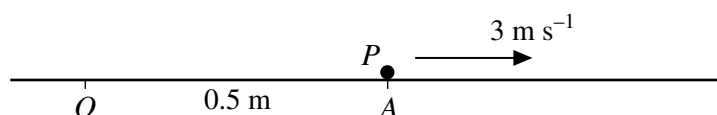
(ii) Calculate the speed of the particle when it is 0.25 m below  $AB$ . [4]

- 5 One end of a light elastic string of natural length 0.3 m and modulus of elasticity 6 N is attached to a fixed point  $O$  on a smooth horizontal plane. The other end of the string is attached to a particle  $P$  of mass 0.2 kg, which moves on the plane in a circular path with centre  $O$ . The angular speed of  $P$  is  $\omega$  rad s<sup>-1</sup>.

(i) For the case  $\omega = 5$ , calculate the extension of the string. [4]

(ii) Express the extension of the string in terms of  $\omega$ , and hence find the set of possible value of  $\omega$ . [4]

6

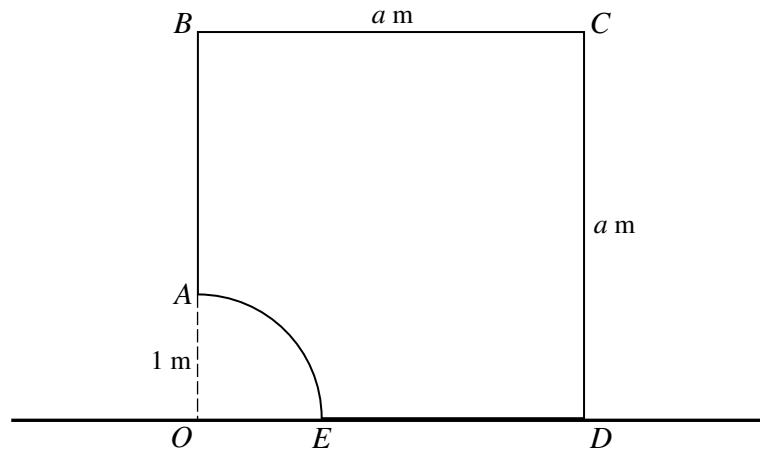


$O$  and  $A$  are fixed points on a horizontal surface, with  $OA = 0.5$  m. A particle  $P$  of mass 0.2 kg is projected horizontally with speed  $3 \text{ m s}^{-1}$  from  $A$  in the direction  $OA$  and moves in a straight line (see diagram). At time  $t$  s after projection, the velocity of  $P$  is  $v \text{ m s}^{-1}$  and its displacement from  $O$  is  $x$  m. The coefficient of friction between the surface and  $P$  is 0.5, and a force of magnitude  $\frac{0.4}{x^2}$  N acts on  $P$  in the direction  $PO$ .

(i) Show that, while the particle is in motion,  $v \frac{dv}{dx} = -\left(5 + \frac{2}{x^2}\right)$ . [2]

(ii) Calculate the distance travelled by  $P$  before it comes to rest, and show that  $P$  does not subsequently move. [7]

[Question 7 is printed on the next page.]



$ABCDE$  is the cross-section through the centre of mass of a uniform prism resting in equilibrium with  $DE$  on a horizontal surface. The cross-section has the shape of a square  $OBCD$  with sides of length  $a$  m, from which a quadrant  $OAE$  of a circle of radius 1 m has been removed (see diagram).

- (i) Find the distance of the centre of mass of the prism from  $O$ , giving the answer in terms of  $a$ ,  $\pi$  and  $\sqrt{2}$ . [5]
- (ii) Hence show that

$$3a^2(2 - a) < \frac{3}{2}\pi - 2,$$

and verify that this inequality is satisfied by  $a = 1.68$  but not by  $a = 1.67$ . [4]